

The response function of a Hall magnetosensor

Y. G. Cornelissens and F.M. Peeters^o

*Departement Natuurkunde, Universiteit Antwerpen (UIA),
Universiteitsplein 1, B-2610 Antwerpen, Belgium*

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Abstract

Patterned two-dimensional electron gas (2DEG) systems into micrometer Hall bars can be used as Hall magnetosensors to provide detailed information on the magnetic field distribution. In this way, ballistic Hall probes have already been studied and used successfully. Here, the response function of a Hall sensor is determined in the diffusive regime, which allows this device to be used as a magnetosensor for the determination of inhomogeneous magnetic field distributions. Furthermore, the influence of the geometry of the Hall bar on this response function, such as circular corners and asymmetry in the probes, is also investigated and appears to be non-negligible.

I. INTRODUCTION

The Hall effect, discovered in 1879 [1], was successfully used to study two-dimensional electron gas (2DEG) systems, which resulted in two Nobel Prizes (in 1985 and 1998, respectively) for the observation of the Integer [2] and the Fractional [3] Quantum Hall Effect. On the other hand, the Hall effect can also be used to provide detailed information on the magnetic field distribution, allowing these 2DEG systems to be used as Hall magnetosensors or Hall sensors [4]. The recent micrometer Hall sensors have considerable advantages compared to other magnetic field measurements, like its noninvasive character, its high magnetic field sensitivity, the small dimensions of the active region and the broad range of temperature and magnetic field strength within which it can be used. Hall probes have previously been used in experiments to study magnetic flux profiles using scanning Hall probe microscopy [5], and were successfully applied for time and space resolved detection of individual vortices in superconductors [6,7]. Furthermore, they are becoming increasingly popular as an alternative for memory devices (MRAM) [8–12]. Recently, submicron ballistic Hall probes were successfully used to investigate the thermodynamic properties of submicron superconducting and ferromagnetic disks [13].

In order to improve the resolution of these Hall magnetometers, it is necessary to provide a quantitative theory which relates the experimental data, in terms of resistance and voltage measurements, to the properties of the magnetic field, and more precisely to the size and strength of the inhomogeneities in this magnetic field. In other words, it is necessary to determine the *Hall response function*. The theory for the Hall magnetometer in the ballistic regime was given in Ref. [14], where it was found that for small magnetic field strength

the Hall resistance is determined by the average magnetic field in the Hall junction ($R_H = \alpha^* \langle B \rangle$, with α^* the effective Hall coefficient), and that it is rather insensitive to the exact position of the magnetic field inhomogeneity. This is in contrast to the diffusive regime, where scattering processes strongly determine the electron transport. In the latter case there is no longer a simple relation between the Hall resistance and the magnetic field inhomogeneity [15], and the Hall resistance depends much more sensitively on the above mentioned factors. Previous studies [14–16] found that the geometry of the device has considerable influence on the Hall response.

The aim of the present paper is to determine the response function of a Hall sensor in the diffusive regime (e.g. this is the regime at room temperature), for given dimensions and geometric characteristics of the device.

This paper is organised as follows. In Sec. II we describe the numerical approach used to determine the Hall response in the diffusive regime, and in Sec. III we give the numerical results for a symmetric Hall cross. The influence of asymmetric geometries of the Hall bar is investigated in Sec. IV, and our conclusions are presented in Sec. V.

II. NUMERICAL APPROACH

In order to describe the transport properties of the 2DEG in the diffusive regime we start from the continuity equation

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0, \quad (1)$$

with \vec{J} and ρ the current and charge density, respectively. In the steady state we have $\partial \rho / \partial t = 0$ and $\vec{\nabla} \times \vec{E} = 0$, which implies that $\vec{E} = -\vec{\nabla} \phi$, with ϕ the potential. For linear transport we use Ohm's law $\vec{J} = \sigma \vec{E}$, where σ is the conductivity tensor, which reduces Eq. (1) to the following two-dimensional partial differential equation $\vec{\nabla} \cdot [\sigma(x, y) \vec{\nabla} \phi(x, y)] = 0$, which can be written more explicitly as

$$\frac{\partial}{\partial x} \left(\sigma_{xx} \frac{\partial \phi}{\partial x} + \sigma_{xy} \frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial y} \left(\sigma_{yy} \frac{\partial \phi}{\partial y} + \sigma_{yx} \frac{\partial \phi}{\partial x} \right) = 0, \quad (2)$$

with $\sigma_{xx} = \sigma_{yy} = \sigma_0 / (1 + [\mu B(\vec{r})]^2)$ and $\sigma_{xy} = -\sigma_{yx} = \mu B(\vec{r}) \sigma_{xx}$, where $\sigma_0 = n_e e \mu$ is the zero field conductivity, with μ the mobility and n_e the electron density of the 2DEG. Eq. (2) is then solved numerically for given boundary conditions using the finite difference method. The boundary conditions reflect the geometry of the Hall bar (Fig. 1), and will vary as different geometries are investigated. The numerical approach presented here is more general than the one presented in Ref. [16], where the effect of probe geometry on the Hall response in the limit of very weak inhomogeneous magnetic fields was studied. Taking the limit of small magnetic fields in Eq. (2), i.e. $\sigma_{xx} = \sigma_{yy} = \sigma_0$ and $\sigma_{xy} = -\sigma_{yx} = \mu B(\vec{r}) \sigma_0$, leads in fact to Eq. (5) of Ref. [16] with $\sigma_0 = \sigma_B$. We found that the use of such a small magnetic field expansion leads to an oversimplification and misses some essential physics of the device.

When a spatially inhomogeneous magnetic field $B(x, y)$ is present the Hall resistance will in general depend on the exact position of the inhomogeneous field distribution with

respect to the Hall cross. This response of the Hall cross is described by a *response function* $F_H(x, y)$ from which we obtain the Hall resistance

$$R_H = -\frac{1}{n_e e} \frac{\int dx \int dy F_H(x, y) B(x, y)}{\int dx \int dy F_H(x, y)}. \quad (3)$$

In the limiting case of a delta-shaped magnetic field profile: $B(x, y) = \Phi \delta(x - x_0) \delta(y - y_0)$, with $\Phi = B_0 S$ the flux through an area S , Eq. (3) leads to the following response:

$$R_H = (-\Phi/n_e e) \frac{F_H(x_0, y_0)}{\int dx \int dy F_H(x, y)}. \quad (4)$$

Consequently,

$$\tilde{F}_H(x_0, y_0) = \frac{F_H(x_0, y_0)}{\int dx \int dy F_H(x, y)} = \frac{R_H}{(-\Phi/n_e e)} \quad (5)$$

is the spatially dependent normalized Hall response of a delta-function magnetic field profile, expressed in units of area^{-1} . In this way it is possible to determine the response function of the Hall cross by placing this delta-function magnetic profile in every point of the cross and calculating the Hall resistance. In practice, a delta function magnetic field distribution can not be used, as it is impossible to generate experimentally, and furthermore, because we have to solve Eq. (2) numerically. Therefore, we shall approximate this profile with a magnetic step of radius r_0 and strength B_0 : $B(x, y) = B_0 \theta(|\vec{r} - \vec{r}'| - r_0)$, which is also called a *magnetic dot*, and which approximates a flux tube of radius r_0 and flux $\Phi = B_0 \pi r_0^2$, centered around position \vec{r}' .

Once the response function is known, one can scan the sample with the Hall sensor and measure the Hall resistance $R_H(x_i, y_j)$ in every point (x_i, y_j) of the sample. The magnetic field distribution $B(x, y)$ is then obtained by performing an inverse transformation. Eq. (3) can be written as follows:

$$R_H(x_i, y_j) = -\frac{1}{n_e e} \int dx \int dy \tilde{F}_H(x, y) B(x_i + x, y_j + y), \quad (6)$$

with (x_i, y_j) the position of the Hall probe on the surface to be scanned (Fig. 2). In polar coordinates this equation becomes

$$R_H(x_i, y_j) = R_H(r_{ij}, \theta_{ij}) = -\frac{1}{n_e e} \int dr \int d\theta \tilde{F}_H(r, \theta) B(r', \theta'), \quad (7)$$

with $r' = \sqrt{r^2 + r_{ij}^2 + 2rr_{ij} \cos(\theta_{ij} - \theta)}$, and $\theta' = \arctan\left(\frac{r \sin \theta + r_{ij} \sin \theta_{ij}}{r \cos \theta + r_{ij} \cos \theta_{ij}}\right)$. Using vector notation we can write the Fourier transform:

$$\begin{aligned} R_H(\vec{k}) &= \int d\vec{r}_{ij} \exp\left(-i\vec{k} \cdot \vec{r}_{ij}\right) R_H(\vec{r}_{ij}) \\ &= \int d\vec{r}_{ij} \exp\left(-i\vec{k} \cdot \vec{r}_{ij}\right) \left\{ \int d\vec{r} \tilde{F}_H(\vec{r}) B(\vec{r} + \vec{r}_{ij}) \right\} \\ &= \int d\vec{r} \exp\left(+i\vec{k} \cdot \vec{r}\right) \tilde{F}_H(\vec{r}) \int d(\vec{r} + \vec{r}_{ij}) \exp\left(-i\vec{k} \cdot (\vec{r} + \vec{r}_{ij})\right) B(\vec{r} + \vec{r}_{ij}) \\ &= \tilde{F}_H(-\vec{k}) B(\vec{k}) \end{aligned} \quad (8)$$

And consequently

$$B(\vec{k}) = \frac{R_H(\vec{k})}{\tilde{F}_H(-\vec{k})}. \quad (9)$$

The magnetic field distribution is then obtained by performing the inverse Fourier transform:

$$B(\vec{r}_{ij}) = \frac{1}{2\pi} \int d\vec{k} \exp(i\vec{k} \cdot \vec{r}_{ij}) B(\vec{k}) = \frac{1}{2\pi} \int d\vec{k} \exp(i\vec{k} \cdot \vec{r}_{ij}) \frac{R_H(\vec{k})}{\tilde{F}_H(-\vec{k})}. \quad (10)$$

III. SYMMETRIC HALL CROSS

The system we envisage is given schematically in Fig. 1, a Hall bar with four identical leads. A voltage drop V_0 across the Hall bar along the y direction generates a current I . The presence of the magnetic dot will then give rise to a potential profile such as the one shown in Fig. 3, from which the longitudinal resistance $R_L = V_0/I$ and the Hall resistance $R_H = V_H/I$ can be calculated. In order to allow immediate comparison with any given experimental setup we scale our physical quantities as follows: lengths in units of the probe width W , magnetic field strength in units of the inverse mobility of the 2DEG, μ^{-1} , voltages in units of V_0 , and resistances in units of the zero magnetic field resistivity $\rho_0 = n_e e \mu$. The normalized Hall response function $\tilde{F}_H(x, y)$ is then given in units of $1/W^2$, i.e. the inverse of the Hall junction area.

The response function

$$\tilde{F}_H(x_i, y_j) = \frac{F_H(x_i, y_j)}{\int dx \int dy F_H(x, y)} = R_H / (B_0 \pi r_0^2) \quad (11)$$

is numerically calculated for a magnetic dot placed in every grid point (x_i, y_j) in the Hall bar. As an example we took $\mu B_0 = 0.2$ and $r'_0 = r_0/W = 0.1$. When moving the dot along the middle of the voltage and current probes we obtained a nearly identical response, given in Fig. 4 for both cases ($\theta = 0^\circ$: solid dots; $\theta = 90^\circ$: open dots). Both responses can be very closely represented by the following relation in polar coordinates:

$$\tilde{F}_H(r, \theta) = A \frac{1}{1 + (Cr)^4}, \quad (12)$$

with $(A, C) = (0.4896, 1.267)$ for $\tilde{F}_H(r, \theta = 0^\circ)$ (dotted curve in Fig. 4), and $(A, C) = (0.4896, 1.303)$ for $\tilde{F}_H(r, \theta = 90^\circ)$ (dashed curve in Fig. 4). Along $(r, \theta = 0^\circ)$ and $(r, \theta = 90^\circ)$ the response is, within 2%, constant over the range $r < 0.2W$. Near the edge of the Hall cross and inside the probes we find a very rapid decrease of \tilde{F}_H .

However, when calculating the Hall response for a magnetic dot placed in every grid point of the Hall cross it is clear from the contourplot in Fig. 5 that this rather simple relation (12), can not be upheld when the angle θ of the cross-section is different from 0° or 90° . Notice that the response function is slightly skewed which was not reproduced by

the linear (in magnetic field) theory of Ref. [16]. As can be seen from Fig. 5, the Hall response function exhibits inverse symmetry with respect to the center of the cross, but is not axially symmetric, as for $\theta = 135^\circ$ two maxima appear at opposite corners of the junction at $r/W \approx 0.5$. We found that this asymmetry is also present for small magnetic fields, and disappears only for extremely low fields, i.e. $B_0 \rightarrow 0$. The broken $x - y$ symmetry arises due to the choice of specific probes which are used for injection of the current, in combination with the Lorentz force.

Next, considering only the central circular part of the junction ($r/W \leq 0.5$), the dot is moved along different angles θ . Choosing at first only the ‘interesting’ directions such as $\theta = 0^\circ, 45^\circ, 90^\circ$ and 135° , indicated in Fig. 5, we obtained the response given in Fig. 6. These curves could be fitted over the range $0 \leq r/W \leq 0.5$ to the expression

$$\tilde{F}_H(r, \theta) = A \frac{1 + (D(\theta)r)^2}{1 + (C_2(\theta)r)^2 + (C_4(\theta)r)^4}, \quad (13)$$

with $D(\theta)$, $C_2(\theta)$ and $C_4(\theta)$ parameters which are a function of the angle θ . The values of these parameters are given in Fig. 7, as a function of the angle θ , along with their calculated values for other angles in the range of $\theta = [0^\circ, 180^\circ]$ (solid symbols). Notice that parameter $C_2 = 1$ for $0^\circ < \theta < 90^\circ$ and approaches zero, i.e. $C_2 = 0$ in the range $90^\circ < \theta < 180^\circ$ which is explained by the curvature of the Hall response for these angle values. This difference in curvature can also be seen from the values for $D(\theta)$ which also exhibit a discontinuity for $\theta > 90^\circ$. Furthermore, parameter C_4 exhibits two distinct minima for $\theta = 45^\circ$ and $\theta = 135^\circ$, the two diagonal directions in the Hall junction. Knowing all these parameter values (A, D, C_2, C_4) for an arbitrary direction θ allows us to describe the Hall response in the central part of the junction with a single expression (Eq. (13)) which, after Fourier transformation, can be inserted in Eq. (10) to calculate the magnetic field distribution.

The parameter A is independent of the direction θ , but is a function of the strength and size of the magnetic dot which we investigated simply by placing a magnetic dot with variable strength and size in the center of the Hall junction. The dependence of the Hall response, in the center of the Hall cross, on the strength μB_0 and the radius r'_0 of the magnetic dot is shown in Fig. 8 (solid dots). The numerical results are closely described by the following relation (solid curves in Fig. 8):

$$A = \tilde{F}_H(0, 0^\circ) = \frac{A_1(r'_0)}{1 + (\mu B_0 * A_2(r'_0))^2}, \quad (14)$$

where the dependence of the parameters A_1 and A_2 on the radius r'_0 is shown in the inset of Fig. 8 (solid and open dots, respectively). In this figure we see that $A_1 = 0.493$ remains constant up to $r'_0 \approx 0.3$ (dotted line), while the parameter A_2 can be represented by a linear equation: $A_2 = 0.522 - 0.229 r'_0$ (dashed line), likewise up to $r'_0 \approx 0.3$. Notice that for $\mu B_0 < 0.5$ the response in the center of the Hall cross ($\tilde{F}_H \approx 0.49$) does practically not depend on the magnetic field strength, nor on the radius of the magnetic dot for $r'_0 \leq 0.4$. Notice also that at $\mu B_0 \approx 0.5$ the curves cross, and consequently that, with increasing radius r'_0 , the Hall response decreases for $\mu B_0 < 0.5$ and increases for $\mu B_0 > 0.5$.

Calculating the Hall response for a weaker magnetic field ($\mu B_0 = 0.1$) we find qualitatively identical results. In Fig. 9 we see two maxima along $\theta = 135^\circ$, the only difference

being that the area of increased Hall response is somewhat smaller in comparison with the previous results for $\mu B_0 = 0.2$. For much stronger magnetic field ($\mu B_0 = 1.0$) we find that there are still two maxima for $\theta = 135^\circ$ with an increased Hall response. It is also clear from Fig. 10 that the difference in response within the junction is much larger for such large magnetic fields.

Finally, we investigated the effect of circular corners in the Hall junction, as this is also present in every experimental setup due to the limited resolution of lithographic techniques with which these devices are fabricated. To study this influence we considered circular corners with different radii $a/W = 0.0, 0.1, 0.2, 0.3$ (see the inset of Fig. 11). In Fig. 11 the Hall response is given along the center of the voltage probes, and it is clear that circular corners in the Hall bar decrease the Hall response considerably. The response function \tilde{F}_H decreases with increasing radius a/W near the center of the Hall cross and \tilde{F}_H stays flat over a larger region in the center. This is a consequence of the increased effective area of the Hall cross. For small a/W we found that in the center of the Hall cross the dependence on the smoothness of the corners can be approximated by $\tilde{F}_H(a) = \tilde{F}_H(0)$. Note also that $\tilde{F}_H = F_H/W^2$, where W^2 was the Hall junction area in the case of square corners. For circular corners this surface area should be replaced by $\tilde{W}^2 > W^2$, where \tilde{W} is an increasing function of the radius a/W . Scaling the response function accordingly, i.e. $F_H/\tilde{W}^2 = \tilde{F}_H \cdot (W^2/\tilde{W}^2)$, would lower the response function even more. But we can conclude from this study that for small radius, i.e. $a/W = 0.1$, which is often a reasonable approximation for actual Hall magnetosensors, the effect of circular corners on the response function remains small, i.e. $< 3\%$. The contourplot of the Hall response for this geometry is given in Fig. 12, for the case of $a/W = 0.2$, where the resemblance with the square geometry in Fig. 5 is apparent. Consequently, the same analytical expression as for the case of square corners, i.e. Eq. (13) can be used. The corresponding parameters are given in Fig. 7 by the open symbols. Notice that only for the parameter C_4 major deviations (with respect to the square corner result) are found for $\theta \approx 45^\circ$ and $\theta \approx 135^\circ$ where the sharp dips are smoothed out.

IV. ASYMMETRIC HALL CROSS

The above results indicated that the sensitivity of a Hall bar is not constant throughout the Hall cross. Thus, one can ask oneself whether it is possible to enhance the sensitivity in certain parts of the Hall cross by using a special cross geometry. Therefore, we investigated the influence of asymmetry of the Hall cross, i.e. when the probes do not have the same width, on the Hall response.

A. Narrow voltage probes

When the geometry of the Hall cross is such that the current probes with width W_C are wider than the voltage probes with width W_V ($W_V < W = W_C$) we obtained a response function which can be substantially larger than $\tilde{F}_H > 0.5$, indicating that a more sensitive region is created by narrowing the voltage probes. The Hall response along the center of the voltage probes is given in Fig. 13 for different values of the voltage probe width: $W_V/W_C = 0.3, 0.5, 0.7, 1.0$. We took $\mu B_0 = 0.2$ and $r'_0 = 0.1$. From this figure it is clear

that a more pronounced peak structure arises as the voltage probe width is decreased. The contourplot for \tilde{F}_H of the geometry with $W_V/W_C = 0.7$ is given in Fig. 14. Notice that the response function was scaled as $\tilde{F}_H = F_H/W^2$ with $W = W_C$. If we scale the response function with the effective Hall junction area $W_C W_V < W^2$ this would lead to an even higher response function. From this we may conclude that it is not necessary to narrow all the probes, but that it is sufficient to do this only for the voltage probes in order to enhance the sensitivity of the device.

B. Asymmetric voltage probes

We also investigated the situation when only one voltage probe is narrow ($W'_V < W = W_V = W_C$) and all other voltage and current probes have the same width. The result is a single-peak function along the voltage probes, given by the curves in Fig. 15, where a more sensitive area is created in the Hall junction close to the narrow probe. For comparison, we also included the results for the other geometries. Compared to the previous situation this geometry leads to a somewhat lower value for the Hall response, but has considerable experimental advantage as only one of the probes would have to be narrowed to obtain a sensitive region. The contourplot of the Hall response for this configuration is given in Fig. 16.

C. Narrow current probes

Finally, also the current probes were narrowed with respect to the voltage probes ($W_C < W = W_V$). As we can see in Fig. 17 the Hall response increases with decreasing current probe width leading to a response function which can be much larger than $\tilde{F}_H > 0.5$, and indicating increased sensitivity of the sensor. In Fig. 18 the contourplot of the Hall response is plotted for this geometry with $W_C/W_V = 0.7$ and we see that, analogous to the case where the voltage probes were narrowed, there are two maxima which are shifted towards the center of the narrowed probe.

V. CONCLUSION

In contrast to the ballistic regime (Ref. [14]) where the Hall response function is a step function $\tilde{F}_H(x, y) = \frac{1}{W^2} \theta(W/2 - |x|) \theta(W/2 - |y|)$, in the diffusive regime it is a smooth function which is constant only near the center of the Hall cross and a decaying function which is asymmetric and different from zero, although small, in the voltage and current probes. From our study it is clear that different regions in the Hall bar are more, or less, sensitive to the presence of a magnetic field. In order to quantify this observation we determined the response function of the Hall device. Lithographic fabrication techniques have a limited resolution which led us to investigate the effect of circular corners in the Hall junction. It was then observed that circular corners decrease the Hall response significantly so that considerable attention should be paid to the resolution with which these devices are fabricated. Bearing in mind the fabrication of Hall sensors, several other geometric

influences were also investigated, and led to the conclusion that more sensitive areas can be created in the Hall junction simply by narrowing the voltage or current leads. Further experimental simplification can be obtained by narrowing only one of the voltage probes as it appeared that this already creates a more sensitive region in the junction.

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FIGURES

FIG. 1. The Hall cross geometry. The current is injected along the y direction.

FIG. 2. The Hall sensor scans the surface (grey area), i.e. it moves across the surface to determine the magnetic field distribution.

FIG. 3. Potential profile arising due to the presence of a magnetic dot in the center of the junction (indicated by the shaded circle) with strength $\mu B_0 = 5.0$ and radius $r'_0 = r_0/W = 0.3$. The values on the contourplot are in units of V_0 , the applied voltage. The current is along the y direction.

FIG. 4. The Hall response for a magnetic dot ($\mu B_0 = 0.2$ and $r'_0 = 0.1$) displaced along the center of the current ($\theta = 90^\circ$) and voltage ($\theta = 0^\circ$) probes. The curves represent Eq. (12) for the corresponding angles.

FIG. 5. Contourplot of the Hall response \tilde{F}_H in a Hall bar. The scanning tip consists of a magnetic dot with strength $\mu B_0 = 0.2$ and radius $r'_0 = 0.1$.

FIG. 6. The Hall response for a magnetic dot ($\mu B_0 = 0.2$ and $r'_0 = 0.1$) displaced along the axis at an angle $\theta = 0^\circ, 45^\circ, 90^\circ$ and 135° with the x axis.

FIG. 7. The values of the parameters in Eq. (13) as a function of θ for a Hall cross geometry with square corners (solid symbols) and circular corners with $a/W = 0.2$ (open symbols).

FIG. 8. Dependence of the Hall response on the strength μB_0 and the radius $r'_0 = r_0/W$ of the magnetic dot. The curves shown are for $r'_0 = 0.05, 0.10, 0.15, 0.2, 0.3, 0.4, 0.5$. The inset shows the dependence of the parameters A_1 and A_2 (Eq. (14)) on the radius of the dot. For small radii ($r'_0 < 0.3$) there exists a linear relation as shown schematically by the dotted and dashed lines, respectively.

FIG. 9. Position dependence of the Hall response \tilde{F}_H in a Hall bar, as resulting from scanning a magnetic dot with strength $\mu B_0 = 0.1$ and radius $r'_0 = 0.1$.

FIG. 10. Contourplot of the Hall response \tilde{F}_H in a Hall bar, due to the presence of a magnetic dot with strength $\mu B_0 = 1.0$ and radius $r'_0 = 0.1$.

FIG. 11. Circular corners in the Hall junction decrease the Hall response. The response function is shown here as resulting from a magnetic dot ($\mu B_0 = 0.2$ and $r'_0 = 0.1$) which is displaced along the center of the voltage probes for different values of $a/W = 0.0, 0.1, 0.2, 0.3$.

FIG. 12. Contourplot of the Hall response \tilde{F}_H in a Hall bar with circular corners ($a/W = 0.2$). The scanning tip consists of a magnetic dot with strength $\mu B_0 = 0.2$ and radius $r'_0 = 0.1$.

FIG. 13. A Hall cross geometry with narrow voltage probes, with respect to the current probes ($W_V < W_C$), creates two much more sensitive regions in the Hall junction, which can be seen from this two-peak function along the center of the voltage probes for $W_V/W_C = 0.3, 0.5, 0.7, 1.0$. The Hall response is scaled as follows $\tilde{F}_H = F_H/W^2$ with $W = W_C$.

FIG. 14. Contourplot of the Hall response \tilde{F}_H in a Hall bar with narrow voltage probes ($W_V/W_C = 0.7$). The scanning tip is a magnetic dot with strength $\mu B_0 = 0.2$ and radius $r'_0 = 0.1$.

FIG. 15. One voltage probe is narrowed with respect to the current probes ($W'_V < W_V = W_C$). This results in a single-peak function along the center of the voltage probes (dotted curve). The dashed curve represents the Hall response $\tilde{F}_H = F_H/W^2$ (with $W = W_C$) where both the voltage probes were narrowed $W'_V = W_V < W_C$, while the solid curve represents the symmetric case $W'_V = W_V = W_C$.

FIG. 16. Contourplot of the Hall response \tilde{F}_H in a Hall bar with one narrow voltage probe $W'_V < W_V = W_C$ ($W'_V/W_V = 0.5$). The scanning tip is a magnetic dot with strength $\mu B_0 = 0.2$ and radius $r'_0 = 0.1$.

FIG. 17. A Hall cross geometry with narrow current probes with respect to the voltage probes ($W_C < W_V$) shows increased sensitivity as the width is decreased. Here, the Hall response $\tilde{F}_H = F_H/W^2$ (with $W = W_V$) along the center of the voltage probes is given for $W_C/W_V = 0.3, 0.5, 0.7, 1.0$.

FIG. 18. Contourplot of the Hall response \tilde{F}_H in a Hall bar with narrow current probes ($W_C/W_V = 0.7$). The scanning tip is a magnetic dot with strength $\mu B_0 = 0.2$ and radius $r'_0 = 0.1$.

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